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- A28 B.Se-I The B-A spraw werthe Al Mathematics Hons: Paper-I GROUP A: SET THEORY

Contents: → sets, subsets, Power set, union, intersection. by P(S). Thus

Definition: A set is a well defined collection of distinct objects. A to -: olgmand

1. N= {1,2,3, ---- }, the set of all natural

 $Z = \{0, \pm 1, \pm 2, ----\}$, the set of all integers,

3. 9 = { = 1 : P, 2 \in Z, 2 \tau }, the set of all rational numbers.

Definition: → Let A and B be two sets. If XEA > XEB, then A is said to be subset of B.

Let A= (a) as ----

That means that each element of A is an element of B. 9+ is denoted by ACB.

Example: - Let A = {1,2,3,4} & B = {1,2,3,4,5} then ACB.

numbers.

Equality of sets

Two sets A and B are said to be equal if they have the same elements. In other words, A=Biff ACB & BCA.

Power Set: > Foor a set S, the power set of s is defined to be the family of all subsets of s. It is denoted by P(S). Thus

Definition: A sed is a well defined collection

Example: - Let A = {a,b,c} . Then arter to $P(A) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \} \}$

THEOREM: A set, containing n elements, the that exactly 2h subsets.

Poroof: Let A = (a1, a2,, and containing De elements, 8 bro A tol constituted

clearly the null set of is a subset of the set A.

since Number of subsets of A containing one elements viz, dais, fait, ---, lant = nc

Number of subsets of A containing two elements = hc, 82 A month

each containing three elements of so on.

The number of subsets of A containing n elements = nCnHence the total number of A subsets of $A = nC_0 + nC_1 + nC_2 + \cdots + nC_n$ $= (1+1)^n = 2^n$ proved.

Example (1) A If $A = \phi$. Then $P(A) = \{\phi\} \quad \{(0)\} = 851A$

Example @ If the set A contains he elements.

then show that P(A) contains 2h elements.

Union and Intersection of Sets:

Let A and B be two sets. The Union of A and B is the set defined by

AUB={x: xEB}

That is, XE AUB if and only if XEA or XEB.

The intersection of A and B is the set defined by

 $AnB = \{x : x \in A \text{ and } x \in B\}$

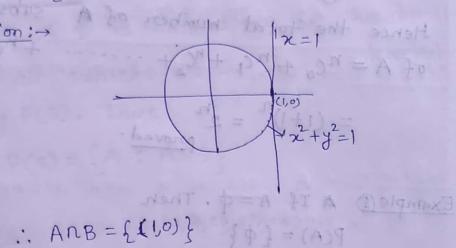
That is, XEARB if and only if XEA and XEB.

Excep.

Exercise 1. If ACB, then find AUB and ARB. Solution: - AUB=B & ARB=A

Ex. z: + Let A := \((x,y) \in R^2: x^2 + y^2 = 1\) and B= (0xy) eR2: x=13. Find AnB.

Solution: >



Ex:3: + Let A be the set of nxh real symmetric matrices, & B the set of nxn real skew-symmetric matrices. Find ARB, (TRY YOURSELF)

Union of A and B is the set defined by

AUB= {x: x GA or x 6B}

that is a e Aus it and only if ach or estant

The intersection of A and B is the set

ANB= [x:xEA and xEB]

That is, xEARB if and only if xEA and xeB.

B.Sc.-I

MATHEMATICS HONS: Paper-I

Goroup A: SET THEORY

Contents: - Complement of a set, De Morgan's Law.

\$ (8 n B)}

(ARB) = A UB

Complement of a set: > Whenever we consider a set, its elements are chosen from some set which we call the universe or the universal set.

If A is a part of the universe U, that is, if ACU, then the complement of A (in U), written as AC, is defined to be

A:= U|A = {x ∈ U : x ¢ A }

Example: \rightarrow Let $U = \{a, b, c, d, e\}$ $A = \{a, e\}$ Then AC = U/A = {xeU; xe & A} :. Ac = { b, c, d} (8dy) 39 (Mak) \$

なる THEOREM: > (De Morgan's Law) . 9 taptoiseA Let A and B be subsets of a Universal set U. Then @ (ARB) = ACUBC (AUB) = ACRBC Proof@: → We need to show that (AnB) C A UBC and A UBC = (AnB). Let oce (ANB)C ⇒fx∈U: x ¢(ANB)} TORESO: TO A STORE BOOK $\Rightarrow \{x \in U: x \notin A \text{ or } x \notin B \text{ or } x \notin A \text{ and } B\}$ > (xev: xex or xeBc) \$ \x \in U; \x \in A^c } or \{ > c \in U; \x \in B^c } * ACCESO > XE(ACUBC) (AnB) C(ACUBC) - D Let y ∈ (ACUBC) A so solition (Uni) A AND CONTRACTOR OF THE STATE OF > SyEU: YEA or YEBC } ⇒ {y∈U: y ∉ A andy ∉ B }

 $\Rightarrow \{ j \in U : j \notin A \text{ and } j \notin B \}$ $\Rightarrow \{ j \in U : j \notin (APAB) \}$ $\Rightarrow \{ j \in U : j \in (APAB) \}$ $\Rightarrow \{ j \in U : j \in (APAB) \}$ $\Rightarrow \{ j \in U : j \in (APAB) \} \}$ $\Rightarrow \{ j \in (APAB) \}$

.: (ACUBC) C (ANB) C_

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From Od @ we get
    (ANB) = A UBC proved.
  Proof (1) :- We need to show that
      (AUB) CACUBC & ACUBC (AUB)
    Let x e & (AUB) > A
too xot > {xeV: xe(AUB)C}
     ⇒ fxeu: x ¢(AUB)}, ylimpt out sol
     ⇒ {x∈U: x & A and x &B}
  etor to fine part and x e Bez
   2N > { XEV: XEANBC}
e that each of the Bant A sixtle family
 (AUB) C ACNB 3
    Let ye(ACNBC)
     ⇒ jy∈U: y∈A and y∈Bc}
     => {y \in U: y \in A and y \in B}
A { JEU: J & AUB}
 et et et je (AUB)C}
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From equation (3) & 4

(AUB) = A RB proved.

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Family of sets: >

Suppose Δ is a non-empty set, and for each $\alpha \in \Delta$ there is a sef $A\alpha$. Then, we have a family of sets indexed by Δ which is written as

{Ax: < A } A > > tol

Here the set 1 is called the index set for the family.

Union of family of sets: ->

Let $\{F_a: x \in \Lambda\}$ be a family of sets indexed by a non-empty set Λ . Let us assume that each of the sets in the family is a subset of some universal set V.

The union of family of sets is defined as

 $UF_{\alpha} = \{x \in U : \exists \alpha \in \Lambda \text{ such that } x \in F_{\alpha}\}$

Intersection of family of sets:

Let $\{F_x: x \in A\}$ be a family of sets indexed by a non-empty set A. Let us assume that each of these sets is a subset of some universal set V. Then the intersection of this family is defined by

 $\prod_{\alpha \in \Lambda} = \left\{ x \in U : \forall \alpha \in \Lambda, \alpha \in F_{\alpha} \right\}$ $\frac{\partial}{\partial x \in \Lambda} = \left\{ x \in U : \forall \alpha \in \Lambda, \alpha \in F_{\alpha} \right\}$

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B.SC-I

Mathematics hons: Paper-I Geraup A: SET THEORY

Contents: - Greneralised De Morgan's Law, Oreneralised distributive law, Creneralised Associative law. (An) → x (An)) = x follows

Remark: > Let {A: : ieI} be an indexed family of subsets of the universal set U. Then UAi = {xeU | xeAi for at least one ieI} &

NAi = {xeu|xeAi, for all ieI}

Oreneralised De Morgan's law: If {Ai: ieI} be an index family of subsets of the universal set U. Then

(i) (YAi) = (Ai anitation de la company)

(ii) $\left(\bigcap_{i} A_{i}\right)^{c} = \bigcup_{i} A_{i}$

Poroofii): > Let x be any element of U (universal set) e Let x∈(VAi)

⇔ x ¢ yAi

⇔ x \ Ai, for any ieI

⇒ x ∈Ai for all ieI

Axe U Vi

and NAi C (NAi)

(VAi) = NAi proved

(VAi) = NAi proved

Proof(ii): → Let x be any element of the universal set & U.

Let x∈(nAi) ⇔x¢nAi

Ax & Ai for at least one iEI.

B { Isi and know that Sic | U ax } = IAU

 $(\Lambda Ai)^{c} \subseteq \Lambda Ai$ and $\Lambda Ai \subseteq (\Lambda Ai)^{c}$

·· (n Ai) = U Ai proved.

Greneralised distributive law:

If [Ai: ie] be an

indexed family of subsets of the universal set U and BCU then

co x & Ai, for any ice I

(i) $BU(\eta Ai) = \eta(BUAi)$

(ii) Bn(YAi) = y(BnAi)

A x e i v

Poroofcis: + Let or be any element of the universal BU(NAi) ((Ay)) = se so d = se | U = se) = {x ∈ BU | x ∈ B or x ∈ (Ñ Ai)} = [xeU|xeB or (xeAi for each ieI)] = {xeV (xeBor xe Ai) for each ie]} = {x ∈ U | x ∈ (BUAi) for each ie]} = n (BUAi) proved. Proofii: * Bn(yAi)
= {xeU|xeB and xe(yAi)} = {xeV|xeB and (xeAi for at least one ie])} = [xeV (xeB and xeAi) for at least one ie] = {xeV|xe(BnAi) for at least one zeI} Parad (HUB) U -= U(BNAi) proved, Greneralised Associative law: > If {Ai; ie] } be an indexed family of subset of & V and BCU then (i) BU(YAi) = Y(BUAi) (i) Bn (nAi) = n(BnAi)

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Proof (2): + Let + be my element of +: (i) foored
        BU (YAL)
     = {xeU | xeBor xe(yAi)}
     = \xeV| xeBor (FiveIst. xeAio)}
      = fxeU ] jioeI such that (xeBorxe Aio)}
      = {xeU| ] ioeI such that (xe BUAio)}
      = U(BUAi)
                proved
                         -boweg (BUA) () =
    Provfai: + Bn (nAi)
      = {xell xell xell and xe(nAi)}
= [xeV|xeB and (xeAi for each iel)}
= {xeV | (xeB and xeAi) for each ieI}
 = [xeV| xe (BNAi) for each ieI]
      = n (BnAi) proved.
                       = Y(BRAi) proved:
                Trenoralised Associative law: -
   an indexed family of subset of du and
                  (iAU8) U = (iAy) US (ib
                  (in Bn (nAi) = p(BnAi)
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